

Unraveling in Guessing Games: An Experimental Study

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Consider the following game: a large number of players have to state simultaneously a number in the closed interval $[0, 100]$. The winner is the person whose chosen number is closest to the mean of all chosen numbers multiplied by a parameter p , where p is a predetermined positive parameter of the game; p is common knowledge. The payoff to the winner is a fixed amount, which is independent of the stated number and p . If there is a tie, the prize is divided equally among the winners. The other players whose chosen numbers are further away receive nothing.¹

The game is played for four rounds by the same group of players. After each round, all chosen numbers, the mean, p times the mean, the winning numbers, and the payoffs are presented to the subjects. For $0 \leq p < 1$, there exists only one Nash equilibrium: all players announce zero. Also for the repeated supergame, all Nash equilibria induce the same announcements and payoffs as in the one-shot game. Thus, game theory predicts an unambiguous outcome.

The structure of the game is favorable for investigating whether and how a player's mental process incorporates the behavior of the other players in conscious reasoning. An explanation proposed, for out-of-equilibrium behavior involves subjects engaging in a finite depth of reasoning on players' beliefs about

one another. In the simplest case, a player selects a strategy at random without forming beliefs or picks a number that is salient to him (*zero-order belief*). A somewhat more sophisticated player forms *first-order beliefs* on the behavior of the other players. He thinks that others select a number at random, and he chooses his best response to this belief. Or he forms *second-order beliefs* on the first-order beliefs of the others and maybe n th order beliefs about the $(n - 1)$ th order beliefs of the others, but only up to a finite n , called the n -depth of reasoning.

The idea that players employ finite depths of reasoning has been studied by various theorists (see e.g., Kenneth Binmore, 1987, 1988; Reinhard Selten, 1991; Robert Aumann, 1992; Michael Bacharach, 1992; Cristina Bicchieri, 1993; Dale O. Stahl, 1993). There is also the famous discussion of newspaper competitions by John M. Keynes (1936 p. 156) who describes the mental process of competitors confronted with picking the face that is closest to the mean preference of all competitors.² Keynes's game, which he considered a Gedankenexperiment, has $p = 1$. However, with $p = 1$, one cannot distinguish between different steps of reasoning by actual subjects in an experiment.

There are some experimental studies in which reasoning processes have been analyzed in ways similar to the analysis in this paper. Judith Mehta et al. (1994), who studied behavior in two-person coordination games, suggest that players coordinate by either applying depth of reasoning of order 1 or by picking a focal point (Thomas C. Schelling, 1964), which they call "Schelling salience." Stahl and Paul W. Wilson (1994) analyzed behavior in symmetric 3×3 games and concluded that subjects were using depths of reasoning of orders 1 or 2 or a Nash-equilibrium strategy.

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¹ The game is mentioned, for example, by Hervé Moulin (1986), as an example to explain rationalizability, and by Mario H. Simonsen (1988).

² This metaphor is frequently mentioned in the macroeconomic literature (see e.g., Roman Frydman, 1982).

Both of these papers concentrated on several one-shot games. In my experiments, the decisions in first period indicate that depths of reasoning of order 1 and 2 may be playing a significant role. In periods 2–4, for $p < 1$, I find that the modal depth of reasoning does not increase, although the median choice decreases over time.³ A simple qualitative learning theory based on individual experience is proposed as a better explanation of behavior over time than a model of increasing depth of reasoning. This is the kind of theory that Selten and Joachim Buchta (1994) call a “learning direction theory,” which has been successfully applied in several other studies.

Other games with unique subgame-perfect equilibria that have been explored in the experimental literature include Robert Rosenthal’s (1981) “centipede game,” a market game with ten buyers and one seller studied experimentally by Roth et al. (1991), a public-goods-provision game studied by Vesna Prasnikar and Roth (1992), and the finitely repeated prisoner’s dilemma studied experimentally by Selten and Rolf Stoecker (1986). In the experimental work on the centipede game by Richard McKelvey and Thomas Palfrey (1992) and on the prisoner’s dilemma supergame, the outcomes are quite different from the Nash equilibrium point in the opening rounds, as well as over time. While the outcomes in Roth et al. (1991), Prasnikar and Roth (1992), and my experiments are also far from the equilibrium in the opening round, they approach the equilibrium in subsequent rounds. Learning models have been proposed to explain such phenomena (see e.g., Roth and Ido Erev, 1995).

I. The Game-Theoretic Solutions

For $0 \leq p < 1$, there exists only one Nash equilibrium at which all players choose 0.⁴ All

³ This kind of unraveling is similar to the naturally occurring phenomena observed by Alvin E. Roth and Xiaolin Xing (1994) in many markets in which it is important to act just a little earlier in time than the competition.

⁴ Assume that there is an equilibrium at which at least one player chooses a positive number with positive probability. Let k be the highest number chosen with positive probability, and let m be one of the players who chooses

announcing 0 is also the only strategy combination that survives the procedure of infinitely repeated simultaneous elimination of weakly dominated strategies.⁵ For $p = 1$ and more than two players, the game is a coordination game, and there are infinitely many equilibrium points in which all players choose the same number (see Jack Ochs [1995] for a survey). For $p > 1$ and $2p < M$ (M is the number of players), all choosing 0 and all choosing 100 are the only equilibrium points. Note that for $p > 1$ there are no dominated strategies.⁶ The subgame-perfect equilibrium play (Selten, 1975) does not change for the finitely repeated game.

II. A Model of Boundedly Rational Behavior

In the first period a player has no information about the behavior of the other players. He has to form expectations about choices of the other players on a different basis than in subsequent periods. In the subsequent periods he gains information about the actual behavior of the others and about his success in earlier periods. Therefore, in the analysis of the data I make a distinction between the first period and the remaining periods.

k with positive probability. Obviously, in this equilibrium p times the mean of the numbers chosen is smaller than k . Therefore, player m can improve his chances of winning by replacing k by a smaller number with the same probability. Therefore no equilibrium exists in which a positive number is chosen with positive probability.

⁵ Numbers in $(100p, 100]$ are weakly dominated by $100p$; in the two-player game, 0 is a weakly dominant strategy. The interpretation of the infinite iteration process might be: it does not harm a rational player to exclude numbers in the interval $(100p, 100]$. If this player also believes that all other players are rational, he consequently believes that nobody will choose from $(100p, 100]$, and therefore he excludes $(100p^2, 100]$; if he thinks that the others believe the same, $(100p^3, 100]$ is excluded, and so on. Thus, 0 remains the only nonexcluded strategy based on common knowledge of rationality. If choices were restricted to integers, all choosing 1 is also an equilibrium.

⁶ It is straightforward to show that all choosing 0 and all choosing 100 are equilibria: it does not pay to deviate from 0 (100) if all other players choose 0 (100) and the number of players is sufficiently large. There is no other symmetric equilibrium since with a unilateral small increase a player improves his payoff. Also, other asymmetric equilibria or equilibria in mixed strategies cannot exist for analogous reasons, as in the case $p < 1$.

The model of first-period behavior is as follows: a player is strategic of degree 0 if he chooses the number 50. (This can be interpreted as the expected choice of a player who chooses randomly from a symmetric distribution or as a salient number à la Schelling [1960]). A person is strategic of degree n if he chooses the number $50p^n$, which I will call iteration step n . A person whose behavior is described by $n = 1$ just makes a naive best reply to random behavior.⁷ However, if he believes that the others also employ this reasoning process, he will choose a number smaller than $50p$, say $50p^2$, the best reply to all other players using degree-1 behavior. A higher value of n indicates more strategic behavior paired with the belief that the other players are also more strategic; the choice converges to the equilibrium play in the limit as n increases.

For periods 2–4, the reasoning process of period 1 can be modified by replacing the initial reference point $r = 50$ by a reference point based on the information from the preceding period. A natural candidate for such a reference point is the mean of the numbers named in the previous period. With this initial reference point, iteration step 1, which is the product of p and the mean of the previous period, is similar to Cournot behavior (Antoine A. Cournot, 1838) in the sense of giving a best reply to the strategy choices made by the others in the previous period (assuming that the behavior of the others does not change from one period to the next).⁸

I can also consider “anticipatory learning,” in which an increase in iteration steps is ex-

pected of the other players. Specifically, one can ask whether, with increasing experience, higher and higher iteration steps will be observed. I will show, however, that the modal frequency, polled over all sessions, remains at iteration step 2 in all periods. In Section V-C a quite different adjustment behavior is examined, which does not involve anything similar to the computation of a best reply to expected behavior. Instead of this, a behavioral parameter—the adjustment factor—is changed in the direction indicated by the individual experience in the previous period.

III. The Experimental Design

I conducted three sessions with the parameter $p = 1/2$ (sessions 1–3), four sessions with $p = 2/3$ (4–7), and three sessions with $p = 4/3$ (8–10).⁹ I will refer to these as $1/2$, $2/3$, or $4/3$ sessions, respectively. A subject could participate in only one session.

The design was the same for all sessions: 15–18 subjects were seated far apart in a large classroom so that communication was not possible. The same group played for four periods; this design was made known in the written instructions. At each individual’s place were an instruction sheet, one response card for each period, and an explanation sheet on which the subjects were invited to give written explanations or comments on their choices after each round. The instructions were read aloud, and questions concerning the rules of the game were answered.¹⁰

After each round the response cards were collected. All chosen numbers, the mean, and

⁷ If the mean choice of the others is 50, the number that really comes nearest to p times the mean is a little lower since this player’s choice also influences p times the mean. My interpretation of iteration step 1 is comparable to the definition of secondary salience introduced by Mehta et al. (1994) or the level-1 type in Stahl and Wilson (1994).

⁸ Actually, Cournot behavior in response to an assumed mean choice \bar{x}_{-i} of the other players would not lead to p times the mean, but to

$$p \frac{M-1}{M-p} \bar{x}_{-i}$$

where M is the number of players. However, there is no indication that subjects try to compute this best reply.

Moreover, for M between 15 and 18, the number of subjects in my experiments, the difference between this best reply and p times the mean is not large.

⁹ I use $p = 1/2$, because it reduces calculation difficulties. With $p = 2/3$, I am able to distinguish between the hypothesis that a thought process starts with the reference point 50 and the game-theoretic hypothesis that a rational person will start the iterated elimination of dominated strategies with 100. For $p > 1$, $p = 4/3$ is used to analyze behavior. There are no sessions with $p = 1$; this game is similar to a coordination game with many equilibria, which has already been studied experimentally (e.g., John Van Huyck et al., 1990).

¹⁰ A copy of the instructions used in the experiment may be obtained from the author upon request.

the product of p and the mean were written on the blackboard (the anonymity of the players was maintained). The number closest to the optimal number and the resulting payoffs were announced. The prize to the winner of each round was 20 DM (about \$13). If there was a tie, the prize was split between those who tied. All other players received nothing.¹¹ After four rounds, each player received the sum of his gains of each period and an additional fixed amount of 5 DM (approximately \$3) for showing up. Each session lasted about 45 minutes, including the instruction period.

IV. The Experimental Results

The raw data can be found in Nagel (1993) and are also available from the author upon request. Whereas I use only nonparametric tests in the following sections, Stahl (1994) applies parametric tests to these data and confirms most of the conclusions.

A. First-Period Choices

Figure 1 displays the relative frequencies of all first-period choices for each value of p , separately. The means and medians are also given in the figure. All but four choices are integers. No subject chose 0 in the $2/3$ and $1/2$ sessions, and only 6 percent chose numbers below 10. In the $4/3$ sessions, only 10 percent chose 99, 100, or 1. Thus, the sessions with different parameters do not differ significantly with respect to frequencies of equilibrium strategies and choices near the equilibrium strategies. *Weakly dominated choices*, choices larger than $100p$, were also chosen infrequently: in the $1/2$ sessions, 6 percent of the

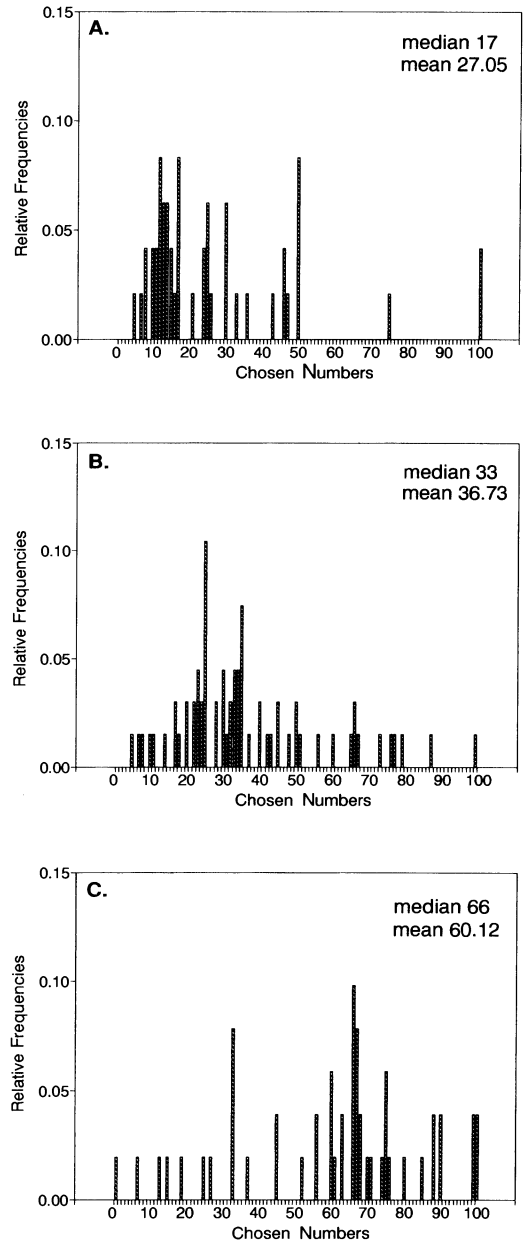


FIGURE 1. CHOICES IN THE FIRST PERIOD: A) SESSIONS 1-3 ($p = 1/2$); B) SESSIONS 4-7 ($p = 2/3$); C) SESSIONS 8-10 ($p = 4/3$)

¹¹ This all-or-nothing payoff structure might trigger unreasonable behavior by some subjects which in turn impedes quicker convergence. In John Duffy and Nagel (1995), the behavior in p -times-the-median game was studied, in an effort to weaken the influence of outliers. While first round behavior in both the mean- and median-treatments was not significantly different, fourth round choices in p -times-the-median game were slightly lower than those in p -times-the-mean game. Changes in the payoff structure, for example, negative payoffs to losers, might affect the evolution of behavior on the guessing game in a different way.

subjects chose numbers greater than 50 and 8 percent chose 50; in the $2/3$ sessions, 10 percent of the chosen numbers were greater than 67, and 6 percent were 66 or 67. From these results

one might infer that either dominated choices are consciously eliminated or reference points are chosen that preclude dominated choices. For $p > 1$, dominated strategies do not exist. Apart from the similarities just mentioned, there are noticeable differences between the distributions of choices in sessions with different values of p . When p was increased, the mean of the chosen numbers was higher. I can reject the null hypothesis that the data from the $1/2$ and $2/3$ sessions are drawn from the same distribution, in favor of the alternative hypothesis that most of the chosen numbers in the $1/2$ sessions tend to be smaller, at the 0.001 level of statistical significance, according to a Mann-Whitney U test. The same holds for a test of the data from the $2/3$ sessions against those of the $4/3$ sessions: the chosen numbers in the former tend to be smaller than those in the latter; the null hypothesis is rejected at the 0.0001 level. This result immediately suggests that many players do not choose numbers at random but instead are influenced by the parameter p of the game.

I also tested whether the data exhibit the structure suggested by the simple model given in Section III, that is, taking 50 as an initial reference point and considering several iteration steps from this point ($50p^n$). Figure 1 shows that the data do not correspond exactly to these iteration steps. However, are the data concentrated around those numbers? In order to test this possibility, I specify *neighborhood intervals* of $50p^n$, for which n is 0, 1, 2, Intervals between two neighborhood intervals of $50p^{n+1}$ and $50p^n$ are called *interim intervals*. I use the geometric mean to determine the boundaries of adjacent intervals. This approach captures the idea that the steps are calculated by powers of n . The interim intervals are on a logarithmic scale approximately as large as the neighborhood intervals, if rounding effects are ignored.¹²

¹² In general, the neighborhood interval of $50p^n$ has the boundaries $50p^{n+1/4}$ and $50p^{n-1/4}$, rounded to the nearest integers, since mostly integers were observed. Note that the neighborhood of $50p^n$ is bounded from the right side by 50 for $p < 1$ and bounded to the left side by 50 for $p > 1$. (The results we present would not change if we had included a right-hand-side neighborhood for $p < 1$, or a left-hand-side neighborhood for $p > 1$).

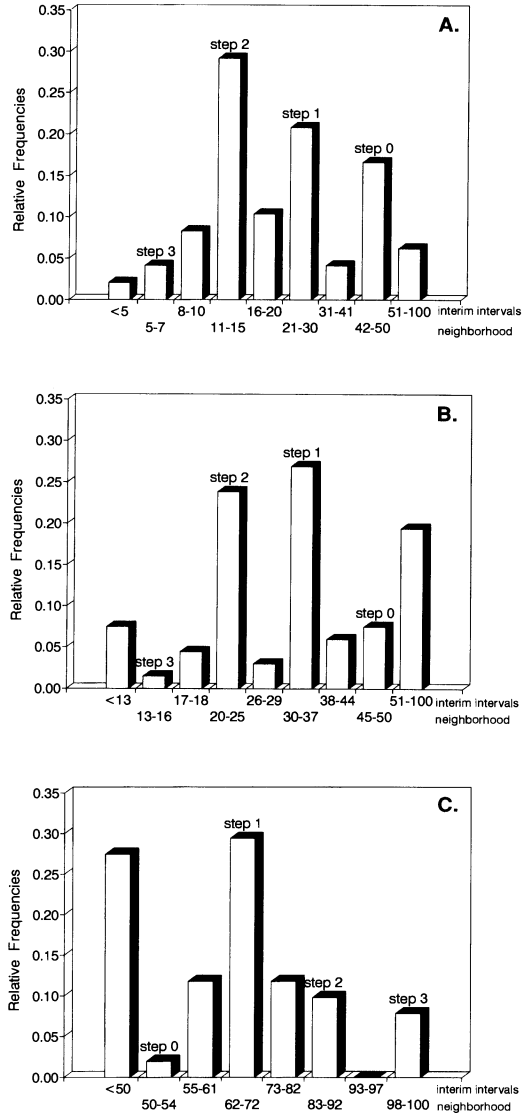


FIGURE 2. RELATIVE FREQUENCIES OF CHOICES IN THE FIRST PERIOD ACCORDING TO THE INTERVAL CLASSIFICATION WITH REFERENCE POINT 50: A) SESSIONS 1–3 ($p = 1/2$); B) SESSIONS 4–7 ($p = 2/3$); C) SESSIONS 8–10 ($p = 4/3$)

Figure 2 shows the number of observations in each of these neighborhood and interim intervals for the respective sessions. The neighborhood and interim intervals are stated on the horizontal axis. Note the similarity between Figure 2A and Figure 2B. In the $1/2$ and $2/3$

sessions, almost 50 percent of the choices are in the neighborhood interval of either iteration step 1 or 2, and there are few observations in the interim interval between them. In all sessions only 6–10 percent are at step 3 and higher steps (the aggregation of the two left-hand columns in Figure 2A and Figure 2B [$p = 1/2$ and $p = 2/3$], respectively, and the right-hand column of Figure 2C [$p = 4/3$]). (Choices above 50 in the $1/2$ and $2/3$ sessions and choices below 50 in the $4/3$ sessions are graphed only in aggregate.) The choices are mostly below 50 in the $1/2$ and $2/3$ sessions and mostly above 50 in the $4/3$ sessions; this difference is statistically significant at the 1-percent level, based on the binomial test.

To test whether there are significantly more observations within the neighborhood intervals than in the interim intervals I consider only observations between step 0 and step 3. Hence, the expected proportion within the neighborhood interval under the null hypothesis (that choices are randomly distributed between interim and neighborhood intervals) is then the sum of the neighborhood intervals divided by the interval between step 0 and step 3. Note that this is a stronger test than taking the entire interval 0–100. The one-sided binomial test, taking into consideration the proportion of observations in the neighborhood intervals, rejects the null hypothesis in favor of the hypothesis that the pooled observations are more concentrated in the neighborhood intervals (the null hypothesis is rejected at the 1-percent level, both for the $1/2$ sessions and for the $2/3$ sessions; it is rejected at the 5-percent level for the $4/3$ sessions).^{13,14}

Note that over all $1/2$ sessions, the optimal choice (given the data) is about 13.5, which

belongs to iteration step 2, which is also where we observe the modal choice, with nearly 30 percent of all observations. Over all $2/3$ sessions, the optimal choice is about 25, which also belongs to iteration step 2 with about 25 percent of all observations, the second-highest frequency of a neighborhood interval. Thus, many players are observed to be playing approximately optimally, given the behavior of the others.

B. *The Behavior in Periods 2, 3, and 4*

To provide an impression of the behavior over time, Figures 3–5 show plots of pooled data from sessions with the same p for each period; the plots show the choices of each subject from round t to $t + 1$. In the $1/2$ and $2/3$ sessions, 135 out of 144 (3 transition periods \times 48 subjects) and 163 out of 201 observations (3×67 subjects), respectively, are below the diagonal, which indicates that most subjects decrease their choices over time. In all sessions with $p < 1$, the medians decrease over time (see Table 1); this is also true for the means except in the last period of the $1/2$ sessions. In the $4/3$ sessions, the reverse is true: 133 out of 153 observations (3×51 subjects) are above the diagonal and the medians increase and are 100 in the third and fourth periods. Thus from round to round, the observed behavior moves in a consistent direction, toward an equilibrium. (It is this movement that is reminiscent of the unraveling in time observed in many markets by Roth and Xing [1994].)

In the $1/2$ sessions, more than half of the observations were less than 1 in the fourth round. However, only three out of 48 chose 0. In the $2/3$ sessions, only one player chose a number less than 1. On the other hand, in the $4/3$ sessions, 100 was already the optimal choice in the second period, being chosen by 16 percent of the subjects; and in the third and fourth period, it was chosen by 59 percent and 68 percent, respectively. Thus, for the $4/3$ sessions I conclude that the behavior of the majority of the subjects can be simply described as the best reply (100) to the behavior observed in the previous period. (Some of the subjects who deviated from this behavior argued that they tried to influence the mean [to bring it

¹³ Since the iterated elimination of dominated strategies starts the reasoning process at 100, I also tested whether that initial reference point would structure the data in a coherent way for the different parameters. For $p = 4/3$ all iteration steps collapse into 100; thus spikes cannot be explained. For the $2/3$ sessions the data are not only concentrated around $100 \times (2/3)^n$. On the other hand, the pattern of the $1/2$ -session data is similar to the pattern in Figure 3A, except that step n becomes step $n + 1$. Hence, 100 is not a plausible initial reference point for most subjects.

¹⁴ The written comments of the subjects also seem to support our model. For details see Nagel (1993).

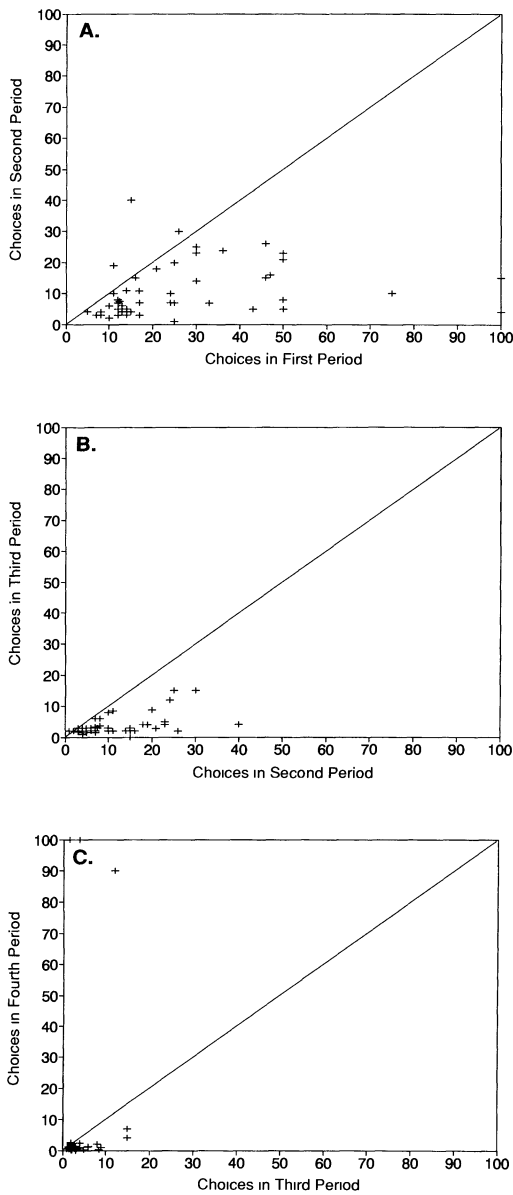


FIGURE 3. OBSERVATIONS OVER TIME FOR SESSIONS 1-3 ($p = 1/2$): A) TRANSITION FROM FIRST TO SECOND PERIOD; B) TRANSITION FROM SECOND TO THIRD PERIOD; C) TRANSITION FROM THIRD TO FOURTH PERIOD

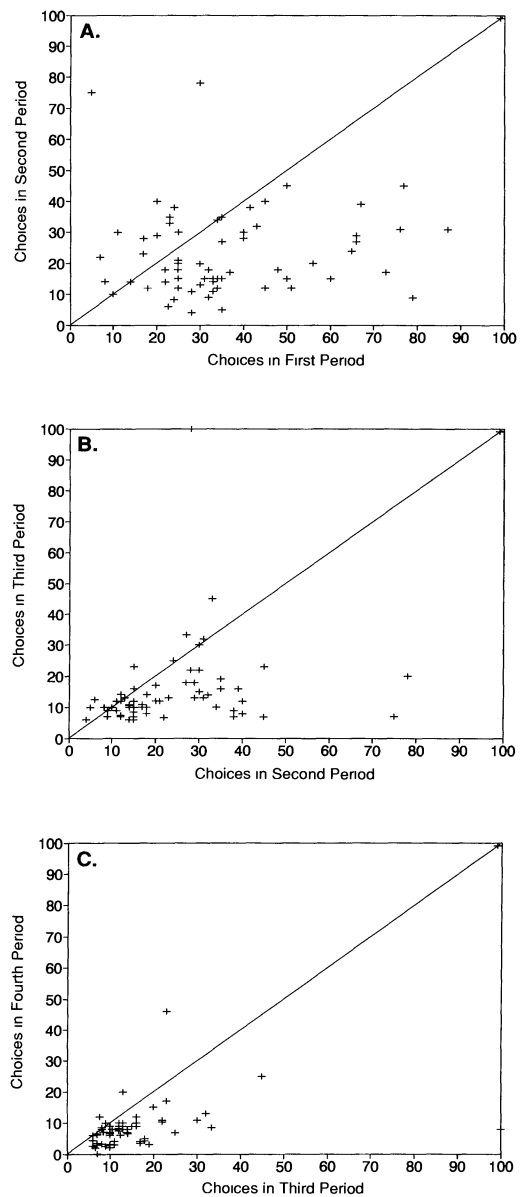


FIGURE 4. OBSERVATIONS OVER TIME FOR SESSIONS 4-7 ($p = 2/3$): A) TRANSITION FROM FIRST TO SECOND PERIOD; B) TRANSITION FROM SECOND TO THIRD PERIOD; C) TRANSITION FROM THIRD TO FOURTH PERIOD

down again] or wrote that the split prize was too small to state the obvious right answer.)

The adjustment process toward the equilibrium in the $1/2$ and $2/3$ sessions is quite different from that in the $4/3$ sessions. Zero is never the

best reply in the $1/2$ and $2/3$ sessions, given the actual strategies. Instead the best reply is a moving target that approaches 0. The adjustment process is thus more complicated. Comparing Figures 3 and 4, one can see that the

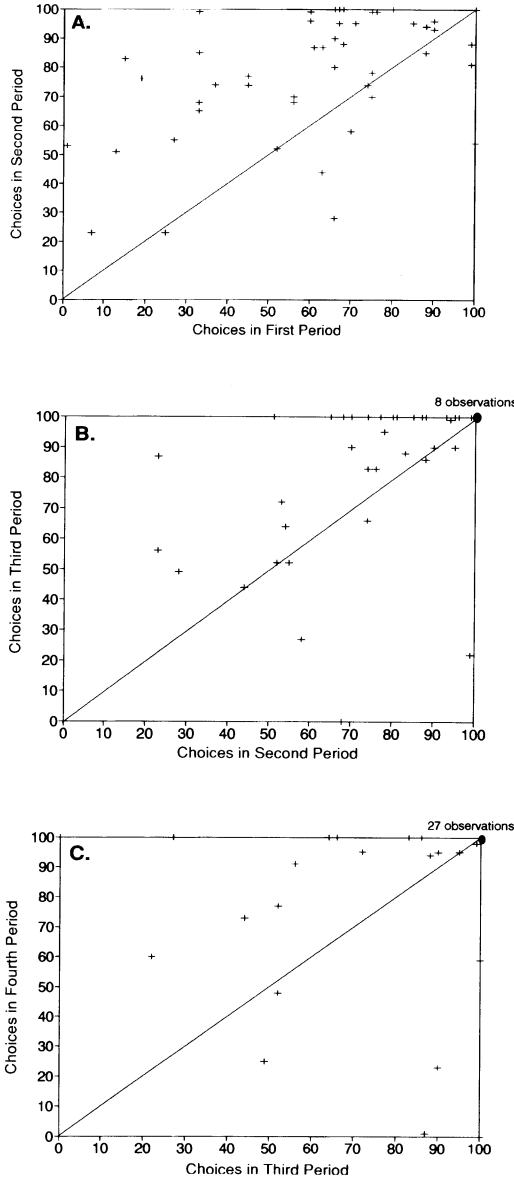


FIGURE 5. OBSERVATIONS OVER TIME FOR SESSIONS 8–10 ($p = 2/3$): A) TRANSITION FROM FIRST TO SECOND PERIOD; B) TRANSITION FROM SECOND TO THIRD PERIOD; C) TRANSITION FROM THIRD TO FOURTH PERIOD

choices in $1/2$ sessions converge faster toward 0 than those in $2/3$ sessions. However, the reason might be that the initial distribution is at lower choices in the former sessions than in the latter. Therefore, to investigate whether the actual choices decrease faster for $p = 1/2$ than

for $p = 2/3$, I define a rate of decrease of the means and medians from period 1 to period 4 within a session by

$$(1a) \quad w_{\text{mean}} = \frac{(\text{mean})_{t=1} - (\text{mean})_{t=4}}{(\text{mean})_{t=1}}$$

$$(1b) \quad w_{\text{med}} = \frac{(\text{median})_{t=1} - (\text{median})_{t=4}}{(\text{median})_{t=1}}$$

The rates of decrease of the single sessions are shown in Table 1, in the last lines of panels A and B. The rates of decrease of the session medians in the $1/2$ sessions are higher than those in the $2/3$ sessions, and the difference is statistically significant at the 5-percent level (one-tailed), based on a Mann-Whitney U test. There is no significant difference in the rates of decrease of the means. The median seems more informative than the mean, since the mean may be strongly influenced by a single deviation to a high number. Thus, I conclude that the rate of decrease depends on the parameter p .

Analyzing the behavior in the first period, I found some evidence that $r = 50$ was a plausible initial reference point. Below, I classify the data of each of the subsequent periods according to the reference point r (mean of the previous period) and iteration steps n : rp^n . Numbers above the mean are aggregated to “above mean, $t-1$ ” (see Table 2).¹⁵

As was the case for the first-period behavior, one cannot expect that exactly these steps are chosen. Grouping the data of the subsequent periods and sessions in the same way as in the first period, namely, in neighborhood intervals of the iteration steps and interim in-

¹⁵ The chosen numbers tend to be below the mean of the previous period, and the difference is significant at the 5-percent level for all $1/2$ and $2/3$ sessions and all periods $t = 2-4$, according to the binomial test. The same test does not reject the null hypothesis for p times the mean of the previous period, for periods 2 and 3. In the fourth period the chosen numbers are significantly (at the 1-percent level) below $p \times r$, in six out of seven sessions. Note that if I had analyzed the data starting from reference point “naive best reply of the previous period” ($p \times r$), instead of starting from the mean, step n would become step $n - 1$, and all choices above the naive best reply would be aggregated to one category.

TABLE 1—MEANS AND MEDIANS OF PERIODS 1–4, AND RATE OF DECREASE FROM PERIOD 1 TO PERIOD 4

A. Sessions with $p = 1/2$:							
Period	Session 1		Session 2		Session 3		Rate of decrease: ^a
	Mean	Median	Mean	Median	Mean	Median	
1	23.7	17	33.2	30	24.2	14	
2	10.9	7	12.1	10	10.2	6	
3	5.3	3	3.8	3.3	2.4	2.1	
4	8.1	2	13.0	0.57	0.4	0.33	
	0.66	0.88	0.61	0.98	0.98	0.97	

B. Sessions with $p = 2/3$:									
Period	Session 4		Session 5		Session 6		Session 7		Rate of decrease: ^a
	Mean	Median	Mean	Median	Mean	Median	Mean	Median	
1	39.7	33	37.7	35	32.9	28	36.4	33	
2	28.6	29	20.2	17	20.3	18	26.5	20	
3	20.2	14	10.0	9	16.7	10	16.7	12.5	
4	16.7	10	3.2	3	8.3	8	8.7	8	
	0.58	0.7	0.92	0.91	0.75	0.71	0.76	0.76	

^a Rate of decrease w from period 1 to 4 (see formula 1).

tervals between two steps, I find no significant difference between the frequencies of observations in the neighborhood intervals and the frequencies of observations in the interim intervals. Note also that as the mean decreases, the interval between two steps becomes rather small.¹⁶ However, I would like to know within which iteration steps the numbers are located in the different periods; therefore, I divide the interval between steps i and $i + 1$ geometrically into two intervals.¹⁷

Parts A and B of Table 2 present the frequencies of observations for each iteration step, pooled over the $1/2$ and $2/3$ sessions, respectively. I also state the mean area of each iteration step over all sessions, separately for each period. In most sessions and periods, at

¹⁶ Most of the subjects just mentioned in their comments that the mean will decrease. There were less precise calculations than in the first period.

¹⁷ If one normalizes the mean of the previous period to 1, the boundaries of step n are $(p^{n+1/2}, p^{n-1/2}]$. As in period 1, step 0 has its right-hand boundary at 1. Table 2 reports the unnormalized length (called "area") of an iteration step. For example, for $p = 1/2$, in period 2, the area of numbers above the mean is 73, since on average, over all $1/2$ sessions the mean of the previous period (r) is 27.

least 80 percent of the observations remain within the bounds of iteration step 0 and iteration step 3, with the modal frequency (30 percent or more) at iteration step 2 when the previous period's mean is the reference point.¹⁸ In fact, in periods 1–3, the best reply is within step 2 in at least five of the seven sessions. Within the neighborhood of the mean of the previous period (step 0) there are only a few observations, and those frequencies decrease in the $2/3$ sessions. The frequency of choices around iteration step 1, corresponding to the Cournot process, also declines to less than 15 percent in the third and fourth periods. The frequencies with more than three steps are below 10 percent, except in period 4 of the $1/2$ sessions. I interpret these results to mean that there is no support for the hypothesis of increasing depth of reasoning, since there is no tendency for the majority of the subjects to increase the depth of reasoning beyond step

¹⁸ This corresponds to what we called the anticipatory learning process in Section II. Hence, one might infer that a substantial proportion of subjects believe that the average behavior in period t will be around p times the mean of period $t - 1$.

TABLE 2—RELATIVE FREQUENCIES AND AREAS OF PERIODS 2–4 ACCORDING TO THE STEP-MODEL FOR AGGREGATED DATA

Classification	Period 2		Period 3		Period 4	
	Relative frequency	Area	Relative frequency	Area	Relative frequency	Area
A. Sessions 1–3 ($p = 1/2$):						
Higher steps	4.2	2.4	4.2	1.0	20.8	0.3
Step 3	25.0	2.4	12.5	1.0	22.9	0.3
Step 2	31.3	4.9	60.4	2.0	29.2	0.7
Step 1	27.0	9.6	12.5	3.9	14.5	1.4
Step 0	2.1	7.9	4.1	3.2	4.2	1.1
Above mean _{<i>t-1</i>}	10.4	73.0	6.3	88.9	8.3	96.2
All	100.0	100.0	100.0	100.0	100.0	100.0
B. Sessions 4–7 ($p = 2/3$):						
Higher steps	7.5	8.9	1.5	5.8	7.5	3.8
Step 3	11.9	4.4	17.9	2.9	25.3	1.9
Step 2	31.3	6.7	46.2	4.3	47.8	2.9
Step 1	20.9	10.0	16.4	6.5	10.4	4.3
Step 0	14.9	6.7	7.5	4.4	3.0	2.9
Above mean _{<i>t-1</i>}	13.4	63.3	10.5	76.1	6.0	84.1
All	100.0	100.0	100.0	100.0	100.0	100.0

2.¹⁹ In the next section I describe the observed behavior from period 2 to period 4 in a different way—by a qualitative learning-direction theory. This theory might explain why the modal frequency of depth of reasoning does not increase.

C. Adjustment Process Due to Individual Experience (for $p < 1$)

So far, I have categorized behavior into classes based on the deviation from the mean of the previous period. I now analyze individual adjustments due to individual experience. There are two possible experiences due to payoffs a player obtained:

- (i) the player gained a share or all of the prize in the previous period, because his announcement was closest to the product of p and the mean; or

- (ii) he earned nothing in the previous period, because his chosen number was either below or above p times the mean (and not the closest to it).

Since there are only a few winners in each period, data on having chosen the winning number are scarce. Therefore, I exclude those choices that led to a positive payoff (19 out of 144 [13 percent] in the $1/2$ sessions and 23 out of 201 [9 percent] in the $2/3$ sessions) and propose a simple qualitative learning theory for the change of behavior after having faced zero payoffs.²⁰

For this purpose, I introduce a parameter called the adjustment factor:

$$(2) \quad a_{it} = \begin{cases} \frac{x_{it}}{50} & \text{for } t = 1 \\ \frac{x_{it}}{(\text{mean})_{t-1}} & \text{for } t = 2, 3, 4 \end{cases}$$

¹⁹ In periods 2 and 3, step 2 is the modal choice in six out of seven sessions; in period 4, this holds in four sessions, and in three sessions, the modal choice is step 3, tied with step 2 or 4.

²⁰ Stahl (1994) compares several learning models. I apply only one learning model, a kind of model that has been successfully used in different experimental settings (see e.g., Selten and Buchta, 1994).

where x_{it} is the number chosen by player i in period t . Hence, a_t is the relative deviation from the mean of the previous period $t - 1$; the mean is the initial reference point. The adjustment factor for period 1 is the choice in period 1 divided by 50, where 50 is the initial reference point, as mentioned in Subsection IV-A. The retrospective "optimal" adjustment factor in period t is defined as the optimal deviation from the mean of period $t - 1$ that leads to p times the mean of period t :

$$(3) \quad a_{opt,t} = \begin{cases} \frac{x_{opt,t}}{50} = \frac{p \times (\text{mean})_t}{50} \\ \text{for } t = 1 \\ \\ \frac{x_{opt,t}}{(\text{mean})_{t-1}} = \frac{p \times (\text{mean})_t}{(\text{mean})_{t-1}} \\ \text{for } t = 2, 3, 4. \end{cases}$$

The idea of this simple learning-direction theory is that in an *ex post* reasoning process a player compares his adjustment factor a_t with the optimal adjustment factor $a_{opt,t}$. In the next period he most likely adapts in the direction of the optimal adjustment factor. Thus, he reflects which deviation from the previous initial reference point would have been better:

$$(4) \quad \begin{aligned} \text{if } a_t > a_{opt,t} &\Rightarrow a_{t+1} < a_t \\ \text{if } a_t < a_{opt,t} &\Rightarrow a_{t+1} > a_t. \end{aligned}$$

In words, if he observed that his chosen number was above p -times the mean in the previous period (i.e., his adjustment factor was higher than the optimal adjustment factor), then he should decrease his rate; if his number was below p times the mean (i.e., his adjustment factor was lower than the optimal adjustment factor), he should increase his adjustment factor.

Figure 6 shows the changes of behavior due to experience from period to period, pooled over all $1/2$ sessions (Fig. 6A-C) and over all $2/3$ sessions (Fig. 6D-F). The bars within each histogram sum to 100 percent. The two left-hand bars in a histogram depict the relative frequencies after the experience that the ad-

justment factor was higher than the optimal adjustment factor, and the two right-hand bars show the frequencies when the factor was lower. The striped bars show the frequencies of *increased* adjustment factors, and the white bars show the frequencies of *decreased* adjustment factors from period t to period $t + 1$.

In each session, pooling the data over the three transition periods, the majority of behavior (between 67 percent and 78 percent, with a mean of 73 percent over all sessions) is in accordance with the learning-direction theory. Thus, taking each session as an independent observation, the null hypothesis that experience is irrelevant can be rejected at the 1-percent level, based on the binomial test. One may also ask whether the frequencies of decreases in adjustment factors independent of experience are higher than the frequencies of increases.²¹ In each session, a majority of subjects decrease the factor; however, the percentage who do so is only between 51 percent and 69 percent, with a mean of 58 percent for all sessions. Comparing the two findings, in each session the frequency in accordance with the learning-direction theory is higher than the frequency of decreases, independent of experience. I interpret this result as indicating that the learning theory provides a better explanation than the hypothesis of decreasing adjustment factor.

The theory of adjustment due to experience is similar to the findings on changes of behavior in other experimental studies. Gerard P. Cachon and Colin Camerer (1991) studied behavior in a coordination game, the so-called *median-effort-game*. They mention that a player who observed that he was below the median in the previous period would most likely increase his effort level and vice versa. Over time, the median effort level remains constant and does not converge to the efficient equilibrium. Also, in Michael Mitzkewitz and Nagel (1993) a simple learning theory related to ours is studied in a completely different setting, with similar results. Selten and Stoecker (1986) analyzed in great detail the influence of experience on end-effect behavior in finite

²¹ This question is related to increasing steps of reasoning.

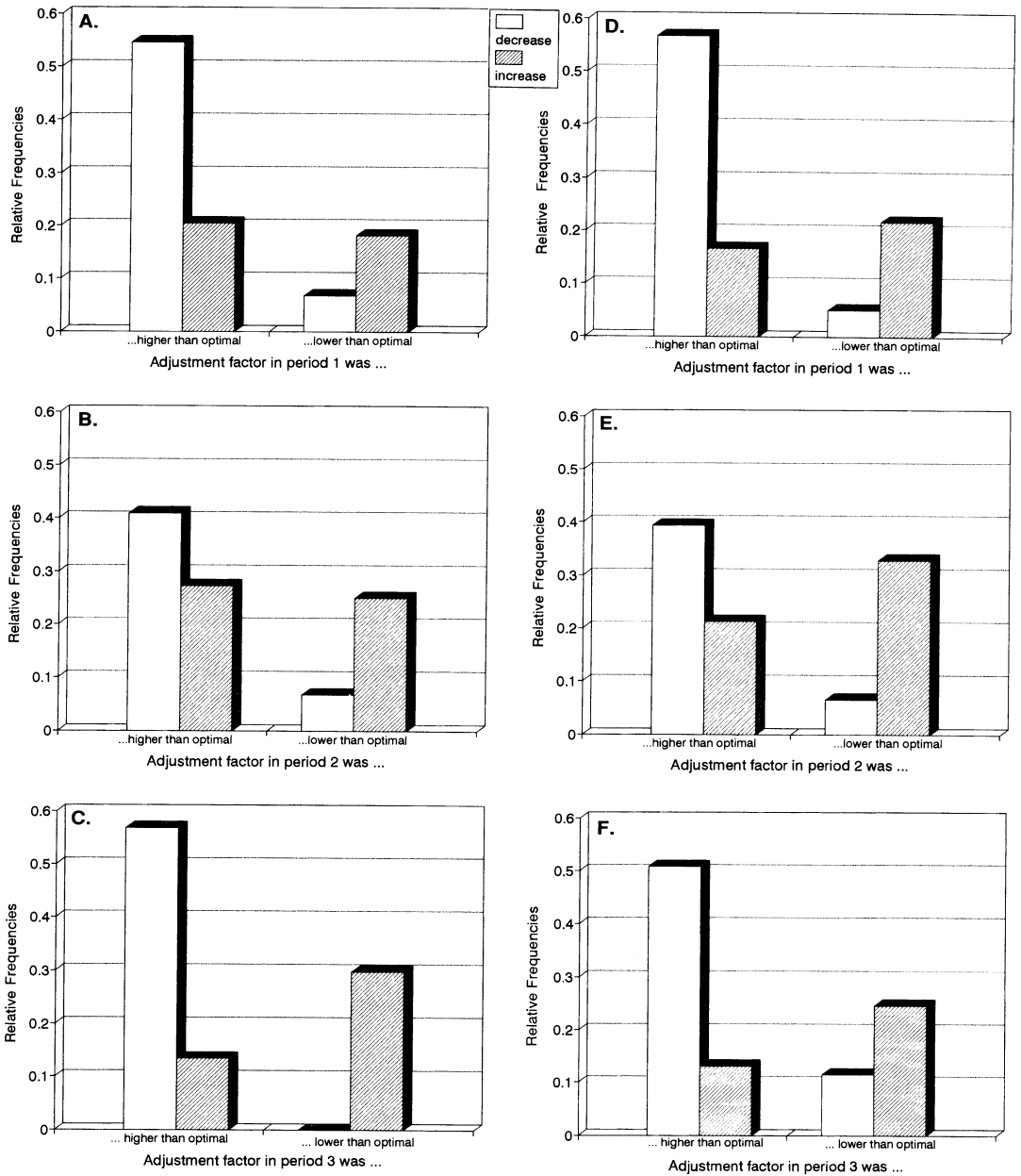


FIGURE 6. RELATIVE FREQUENCIES OF CHANGES IN ADJUSTMENT FACTORS DUE TO INDIVIDUAL EXPERIENCE IN THE PRECEDING PERIOD: A) $p = 1/2$, TRANSITION FROM FIRST TO SECOND PERIOD; B) $p = 1/2$, TRANSITION FROM SECOND TO THIRD PERIOD; C) $p = 1/2$, TRANSITION FROM THIRD TO FOURTH PERIOD; D) $p = 2/3$, TRANSITION FROM FIRST TO SECOND PERIOD; E) $p = 2/3$, TRANSITION FROM SECOND TO THIRD PERIOD; F) $p = 2/3$, TRANSITION FROM THIRD TO FOURTH PERIOD

prisoner's-dilemma supergames. Thus for different games, similar kinds of adjustment processes have been used to explain behavior. However, the dynamics of the behavior can be quite different: in some games there is a convergence toward an equilibrium, whereas in others, the adjustment process may not lead to an (efficient) equilibrium.

V. Summary

My analysis of behavior in an abstract game leads me to believe that the structure of the game is favorable for the study of thought processes of actual players. In the first period the behavior deviates strongly from game-theoretic solutions. Furthermore, the distribution of the chosen numbers over the $[0, 100]$ interval in sessions with different parameters were significantly different. I have proposed a theory of boundedly rational behavior in which the "depths of reasoning" are of importance. The results indicate that, starting from initial reference point 50, iteration steps 1 and 2 play a significant role, that is, most of the observations are in the neighborhood of $50p$ or $50p^2$, independent of the parameter p . This result accounts for the difference of the distributions of the chosen numbers for different parameter values p .

Thus, the theory of boundedly rational behavior for the first period deviates in several ways from the game-theoretic reasoning:

- (i) I suggested that the "reference point" or starting point for the reasoning process is 50 and not 100. The process is driven by iterative, naive best replies rather than by an elimination of dominated strategies.
- (ii) The process of iteration is finite and not infinite.
- (iii) I apply the same theory for $p > 1$ and $p < 1$, whereas game-theoretic reasoning is different for those parameter sets.

Over time the chosen numbers approach an equilibrium or converge to it. In the $4/3$ sessions, the choice 100 is the best reply in all periods but the first. In the third and fourth period more than 50 percent of the subjects choose this strategy. In the sessions with $p < 1$, there is a moving target, which approaches

zero. I apply the theory of first-round behavior also to the subsequent periods 2–4, using as the initial reference point the mean of the previous period. I find that the modal choices are around iteration step 2, and the majority of observations remain below step 3. In most sessions, the best reply is within step 2 in periods 1–3. I cannot accept the hypothesis of increasing iteration steps, and I suggest that another explanation of the observed behavior may be more adequate for periods 2–4. I propose a qualitative learning-direction theory which predicts that a subject tends to increase his adjustment factor in the direction of the optimal adjustment factor if it was below the optimal one and tends to decrease the adjustment factor if it was above the optimal one. A similar kind of simple learning theory has been applied successfully in other experiments.

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